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Abstract

A major lesson of the recent financial crisis is that the ability of banks to withstand liquidity shocks and to provide lending to one another is crucial for financial stability. This paper studies the functioning of the interbank lending market and the optimal policy of a central bank in response to both idiosyncratic and aggregate shocks. In particular, we consider how the interbank market affects a bank's choice between holding liquid assets ex ante and acquiring such assets in the market ex post. We show that a central bank should use different tools to manage different types of shocks. Specifically, it should respond to idiosyncratic shocks by lowering the interest rate in the interbank market and address aggregate shocks by injecting liquid assets into the banking system. We also show that failure to adopt the optimal policy can lead to financial fragility.

Key words: bank liquidity, interbank markets, central bank policy, financial fragility, bank runs

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1 Introduction

The appropriate response of a central bank's interest rate policy to banking crises is the subject of a continuing and important debate. A standard view is that monetary policy should play a role only if a financial disruption directly affects inflation or the real economy; monetary policy should not be used to alleviate financial distress per se. Additionally, several studies on interlinkages between monetary policy and financial-stability policy recommend the complete separation of the two, with evidence of higher and more volatile inflation rates in countries where the central bank is in charge of banking stability.[1]

This view of monetary policy is challenged by observations that during a banking crisis, interbank interest rates often appear to be a key instrument used by central banks for limiting threats to financial stability. During the recent crisis starting in August 2007, interest rate setting in both the U.S. and the E.U. appeared to be geared heavily toward alleviating stress in the banking system. This also appears to be the case in previous financial disruptions, as Goodfriend (2002) states: "Consider the fact that the Fed cut interest rates sharply in response to two of the most serious financial crises in recent years: the October 1987 stock market break and the turmoil following the Russian default in 1998." The practice of reducing interbank rates during financial turmoil also challenges the long-debated view originated by Bagehot (1873) that central banks should provide liquidity to banks at high penalty interest rates (see Martin 2009, for example).

In order to understand the role for monetary policy during banking crises, it is important to have a framework to address the issue in its most basic form. Interbank lending markets are a critical source of external liquidity for banks during financial turmoil, and interbank interest rates are the fundamental instrument of monetary policy. In our paper, we develop a model for studying the role of optimal central bank interest rate policy in interbank markets in the event of both idiosyncratic and aggregate liquidity shocks. We examine whether the interbank market can provide optimal liquidity to banks during a crisis. We question whether access to interbank market liquidity helps or hurts banks' incentives to hold liquid assets internally ex ante, and we ask if central bank policy can help.

Our main results are that 1) an interbank market can be part of an optimal institutional

[1] See Goodhart and Shoenmaker (1995) and Di Giorgio and Di Noia (1999).

arrangement, 2) the central bank can achieve the full-information first best allocation and should use different tools to respond to different types of shocks, and 3) failure by the central bank to follow the optimal policy can lead to financial fragility. In particular, we show that there exists a first best equilibrium in which the central bank sets low interbank rates during idiosyncratic disruptions to enable efficient redistribution of liquidity, sets high rates during non-disruptive times using a symmetric-rate policy to induce banks to hold liquid assets ex ante, and injects additional liquidity into the banking system during aggregate shocks.

Intuition for our results can be gained by understanding the role of banks and the interbank market in our model. Under incomplete markets, a primary role for banks is to provide greater risk-sharing and liquidity to depositors who face uninsurable idiosyncratic liquidity shocks. During financial disruptions, which we think of as states when banks face considerable uncertainty regarding their need for liquid assets, banks themselves may have large borrowing needs in the interbank market. We show that an interbank market can achieve the optimal allocation—allowing banks to provide efficient risk-sharing to their depositors and insuring banks against idiosyncratic liquidity shocks—provided that the interest rate in this market is state dependent and low in states of financial disruption. The need for a state-dependent interest rate suggests a role for the central bank.

In our model, the interest rate on the interbank market plays two roles. From an ex-ante perspective, the expected rate influences the banks' portfolio decision between short-term liquid assets and long-term illiquid assets. Ex post, the rate determines the terms at which banks can borrow liquid assets in response to idiosyncratic shocks. There is a trade-off between the two roles: If the rate expected ex ante is equal to the rate realized ex post in every state, then the efficient allocation cannot be achieved. Indeed, if the rate is low, the redistribution of liquid assets between banks subject to idiosyncratic shocks will be efficient, but banks will choose a suboptimal portfolio. At the rate that induces banks to invest in the optimal portfolio, the interbank market does not achieve an optimal redistribution. If the interbank rate is state dependent, however, the rate expected ex ante does not need to be equal to the rate ex post in every state. A high expected rate can induce banks to hold the optimal portfolio, while a low rate in states of financial disruption allows the efficient redistribution of assets between banks.

A result of our model is the existence of multiple rational expectations equilibria, only

one of which is optimal. This multiplicity of equilibria emerges because of the inelasticity of the demand and supply of funds in the interbank market, which is one of the main features of our model and which highlights the fundamentally inelastic nature of banks' short-term liquidity needs. The interbank market clears for a number of interest rates. Yet, different rationally expected distributions of future interest rates support different allocations as banks choose to invest more or less in the liquid asset. There are two potential sources of inefficiencies. First, banks may invest too much in the long-term illiquid asset and too little in the short-term liquid asset. Second, bank depositors may bear consumption risk caused by idiosyncratic bank liquidity shocks.

The role of the central bank is to implement the efficient allocation by choosing the interest rate in the interbank market contingent on the state, by which it can select the Pareto optimal equilibrium. This result is in line with the view that central banks should disclose their strategies ex ante, as this allows financial markets to allocate resources in a more efficient way. In our model, the rational expectations equilibria take into account the behavior of the central bank. Consequently, a well-defined (contingent) rule on interbank interest setting by the central bank will help coordinate banks.

Our model also illustrates how the central bank can respond to different types of shocks with different tools. The central bank can optimally respond to idiosyncratic liquidity shocks by changing the interbank rate, a standard tool. In contrast, the central bank must inject liquid assets into banks to respond to an aggregate shock. The central bank can achieve the optimal allocation by holding liquid assets on its balance sheet and injecting the liquidity into the banking system in the face of aggregate depositor withdrawal shocks, such that banks can meet their liquidity needs. These liquidity injections, which somewhat resemble fiscal policy, are similar to some of the unconventional tools used by central banks during the recent crisis.

Our paper shows that even though the interbank market is ex-post efficient, idiosyncratic liquidity shocks can cause bank runs if the central bank does not implement the optimal policy of lowering interest rates after shocks. Financial fragility can arise when the interbank rate is high because banks that must borrow liquidity to pay for large idiosyncratic depositor withdrawals will have few resources left for their remaining depositors. If shocks are sufficiently large, the resources available to remaining depositors may be so low that they withdraw before their true liquidity needs arise, causing a bank run. The

optimal central bank policy prescribes low interest rates in states with large idiosyncratic shocks, allowing a better redistribution of resources between banks and eliminating banks' susceptibility to runs.

Despite the importance of interbank markets for financial stability, there are relatively few papers in this field, possibly because there was no theory that had interbank markets as part of an optimal arrangement, until recently. In their seminal study, Bhattacharya and Gale (1987) examine banks with idiosyncratic liquidity shocks from a mechanism design perspective. The constrained-efficient arrangement in their paper shows how setting a limit on the size of individual loan contracts among banks helps incentives and improves efficiency. Our paper, in contrast, shows that an interbank spot market that allows for unlimited borrowing and lending at the market interest rate can achieve the full-information efficient allocation. More recent work by Freixas and Holthausen (2005), Freixas and Jorge (2008), and Heider, Hoerova, and Holthausen (2008) assume the existence of interbank markets even though they are not part of an optimal arrangement.

Both our paper and that of Allen, Carletti, and Gale (2008) develop frameworks in which interbank markets are efficient. In Allen, Carletti and Gale (2008), the central bank responds to both idiosyncratic and aggregate shocks by buying and selling assets, using its balance sheet to achieve the efficient allocation. In our model, the central bank uses a different tool depending on the nature of the shock. The central bank uses its balance sheet to respond to aggregate shocks, but lowers the interbank interest rate to respond to idiosyncratic shocks.

Our model of central bank intervention provides an alternative mechanism to that of Guthrie and Wright (2000) to produce results termed "open mouth operations," which refers to the concept that the central bank can determine short-term real interest rates without active trading intervention in equilibrium. Goodfriend and King (1988) argue that, with efficient interbank markets, monetary policy should respond to aggregate but not idiosyncratic liquidity shocks. We find that despite interbank markets being ex-post efficient, a role for monetary policy is to insure banks against idiosyncratic shocks, which we show can create an inefficient distribution of liquidity among banks. The results of our paper are similar to those of Diamond and Rajan (2008) in showing a benefit to reducing interest rates during a crisis, which leads to moral hazard for banks' choice of liquidity holding and requires a symmetric interest rate policy with high rates in good

times. Diamond and Rajan (2008) examine the limits of central bank influence over bank interest rates based on a Ricardian equivalence argument, whereas we find a new mechanism by which the central bank can adjust interest rates based on the inelasticity of banks' short-term supply and demand for liquidity. Our paper also relates to Bolton et al. (2008) in examining the efficiency of financial intermediaries' choice of holding liquidity versus acquiring liquidity supplied by the market after shocks occur. Efficiency depends on the timing of central bank intervention in Bolton et al. (2008), whereas in our paper the level of interest rate policy is the focus. Acharya and Yorulmazer (2008) consider interbank markets with imperfect competition. Gorton and Huang (2006) study interbank liquidity historically provided by banking coalitions through clearinghouses. Ashcraft, McAndrews, and Skeie (2008) examine a model of the interbank market with credit and participation frictions that can explain their empirical findings of reserves-hoarding by banks and extreme interbank rate volatility.

2 Model

The model has three dates, denoted by $t = 0, 1, 2$, and a continuum of competitive banks, each with a unit continuum of consumers. Ex-ante identical consumers are endowed with one unit of good at date 0 and learn their private type at date 1. With a probability $\overline{\lambda}$, a consumer is "impatient" and needs to consume at date 1. With complementary probability $1 - \overline{\lambda}$, a consumer is "patient" and needs to consume at date 2. Throughout the paper, we disregard sunspot-triggered bank runs.

There are two possible technologies. The short-term liquid technology allows for storing goods at date 0 or date 1 for a return of one in the following period. The long-term investment technology allows for investing goods at date 0 for a return of $r > 1$ at date 2. Investment is illiquid and cannot be liquidated at date 1.[2]

2.1 Liquidity shocks

The banking system may face both aggregate and idiosyncratic liquidity shocks. The aggregate fraction of impatient depositors in the economy can take two values: $\overline{\lambda}_L$, with probability $\pi \in [0, 1]$, and $\overline{\lambda}_H$, with probability $1-\pi$, where $\overline{\lambda}_H > \overline{\lambda}_L$ and $\pi\overline{\lambda}_L + (1-\pi)\overline{\lambda}_H =$

[2] We extend the model to allow for liquidation at date 1 in Section 7.

$\overline{\lambda}$. In addition, at date 1, banks may be affected by an idiosyncratic shock. The state of the world with respect to this shock is indexed by $i \in \mathcal{I} \equiv \{0, 1\}$, where

$$i = \begin{cases} 1 & \text{with prob } \rho \\ 0 & \text{with prob } 1 - \rho. \end{cases}$$

Banks are ex-ante identical at date 0. At date 1, each bank learns its private type $j \in \mathcal{J} \equiv \{h, l\}$, where

$$j = \begin{cases} h & \text{with prob } \frac{1}{2} \\ l & \text{with prob } \frac{1}{2}. \end{cases}$$

Half of banks are type h and half are type l. Banks of type $j \in \mathcal{J}$ have a fraction of impatient consumers at date 1 equal to

$$\lambda_a^{ij} = \begin{cases} \overline{\lambda}_a + i\varepsilon & \text{for } j = h \\ \overline{\lambda}_a - i\varepsilon & \text{for } j = l, \end{cases}$$

where $a \in \mathcal{A} \equiv \{H, L\}$, $i \in \mathcal{I}$ and $\varepsilon > 0$ is the size of the bank-specific idiosyncratic liquidity shock. We assume $0 < \lambda_a^{il} \leq \lambda_a^{ih} < 1$ for $a \in \mathcal{A}$, $i \in \mathcal{I}$. To summarize, when $i = 1$, banks of type $j = h$ have relatively high withdrawals at date 1 and banks of type $j = l$ have relatively low withdrawals. When $i = 0$, all banks face the same withdrawals at date 1. At date 2, banks of type $j \in \mathcal{J}$ have a fraction of patient consumers equal to $1 - \lambda_a^{ij}$, $a \in \mathcal{A}$, $i \in \mathcal{I}$.

At date 0, consumers deposit their unit good in their bank for a deposit contract that pays a noncontingent amount for withdrawal and consumption at date 1 of $c_1 \geq 0$, or pays an equal share of the bank's remaining goods for withdrawal and consumption at date 2 of $c_{2a}^{ij} \geq 0$.[3] Consumer utility is

$$U = \begin{cases} u(c_1) & \text{with prob } \overline{\lambda}, \text{ for impatient depositors} \\ u(c_1 + c_{2a}^{ij}) & \text{with prob } 1 - \overline{\lambda}, \text{ for patient depositors,} \end{cases}$$

[3] A possible justification for the noncontingent payment to impatient consumers combined with the contingent payment to patient consumers could be developed by introducing shareholders. At date $t = 1$, impatient consumers sell their shares to patient consumers in exchange for a fixed payment c_1. For the sake of simplicity, we do not explicitly model shareholders, but some of our results can be reinterpreted in terms of shareholder compensation for higher risk-taking.

where u is increasing and concave. A consumer's expected utility is

$$\begin{aligned} E[U] &= \left[\pi\overline{\lambda}_L + (1-\pi)\overline{\lambda}_H\right] u(c_1) \\ &\quad +(1-\rho)\left[\pi(1-\overline{\lambda}_L)u(c_{2L}^0) + (1-\pi)(1-\overline{\lambda}_H)u(c_{2H}^0)\right] \\ &\quad +\rho\frac{\pi}{2}\left[(1-\lambda_L^{1h})u(c_{2L}^{1h}) + (1-\lambda_L^{1l})u(c_{2L}^{1l})\right] \\ &\quad +\rho\frac{1-\pi}{2}\left[(1-\lambda_H^{1h})u(c_{2H}^{1h}) + (1-\lambda_H^{1l})u(c_{2H}^{1l})\right]. \end{aligned}$$

Banks maximize their depositors' expected utility and make zero profit because of competition for deposits at date 0. Banks invest $\alpha \in [0,1]$ in long-term assets and store $1-\alpha$ in liquid goods. At date 1, consumers and banks learn their private type. Bank j borrows $f_a^{ij} \in \mathbb{R}$ on the interbank market (the notation 'f' represents the fed funds market) and consumers withdraw. At date 2, bank j repays the amount $f_a^{ij}\iota_a^i$ for its loan and the bank's remaining consumers withdraw, where ι_a^i is the interbank lending gross rate of return. Since banks are able to store goods between dates 1 and 2, $\iota_a^i \geq 1$ for all $a \in \mathcal{A},\ i \in \mathcal{I}$.

The bank budget constraints for bank j for dates 1 and 2 are

$$\lambda_a^{ij} c_1 = 1 - \alpha - \beta_a^{ij} + f_a^{ij} \qquad \text{for } a \in \mathcal{A},\ i \in \mathcal{I},\ j \in \mathcal{J} \tag{1}$$

$$(1-\lambda_a^{ij})c_{2a}^{ij} = \alpha r + \beta_a^{ij} - f_a^{ij}\iota_a^i \qquad \text{for } a \in \mathcal{A},\ i \in \mathcal{I},\ j \in \mathcal{J}, \tag{2}$$

respectively, where $\beta_a^{ij} \in [0, 1-\alpha]$ is the amount of liquid goods that banks of type j store between dates 1 and 2. We assume that banks lend goods when indifferent between lending and storing. We also assume that banks cannot contract with each other at date 0. Further, we assume that the coefficient of relative risk aversion for $u(c)$ is greater than one, which implies that banks provide risk-decreasing liquidity insurance. For the baseline model, we consider parameters such that there are no bank defaults in equilibrium.[4] As such, we assume that incentive compatibility holds:

$$c_{2a}^{ij} \geq c_1 \text{ for all } a \in \mathcal{A},\ i \in \mathcal{I},\ j \in \mathcal{J},$$

which rules out bank runs based on very large idiosyncratic shocks.

From the date 1 budget constraint (1), we can solve for

$$f_a^{ij} = \lambda_a^{ij} c_1 - (1-\alpha) + \beta_a^{ij}.$$

[4] Bank runs are considered in Section 6.

Substituting this in the date 2 budget constraint (2) and rearranging gives

$$c_{2a}^{ij} = \frac{\alpha r + \beta_a^{ij} - [\lambda_a^{ij} c_1 - (1-\alpha) + \beta_a^{ij}]\iota^i}{(1-\lambda_a^{ij})}. \tag{3}$$

A bank's optimization to maximize its depositors' expected utility is

$$\max_{\alpha\in[0,1];c_1,\{\beta_a^{ij}\}_{\alpha\in\mathcal{A},i\in\mathcal{I},j\in\mathcal{J}}\geq 0} \quad E[U] \tag{4}$$

$$\text{s.t.} \quad \beta_a^{ij} \leq 1-\alpha \qquad \text{for } a\in\mathcal{A},\ i\in\mathcal{I},\ j\in\mathcal{J} \tag{5}$$

$$(3) \qquad \text{for } a\in\mathcal{A},\ i\in\mathcal{I},\ j\in\mathcal{J}, \tag{6}$$

where the constraint gives the maximum amount of goods that can be stored between dates 1 and 2.

3 The planner's allocation

To find the first best allocation, we consider a planner who can observe consumer types. The planner can ignore idiosyncratic shock i and bank types j and needs to worry only about the aggregate share of impatient depositors in the economy. The planner maximizes the expected utility of depositors subject to feasibility constraints:

$$\max_{\alpha\in[0,1];c_1\geq 0,\beta\geq 0} \quad \pi\left[\overline{\lambda}u(c_1) + \left(1-\overline{\lambda}_L\right)u(c_{2L})\right] + (1-\pi)\left[\overline{\lambda}u(c_1) + \left(1-\overline{\lambda}_H\right)u(c_{2H})\right]$$

$$\text{s.t.} \quad \overline{\lambda}_L c_1 < \overline{\lambda}_H c_1 \leq 1-\alpha+\beta$$

$$\left(1-\overline{\lambda}_L\right)c_{2L} \leq \alpha r + 1 - \alpha - \beta - \overline{\lambda}_L c_1$$

$$\left(1-\overline{\lambda}_H\right)c_{2H} \leq \alpha r + 1 - \alpha - \beta - \overline{\lambda}_H c_1$$

$$\beta \leq 1-\alpha.$$

The constraints are the physical quantities of goods available for consumption at date 1 and 2 and available for storage between dates 1 and 2, respectively.

If there are no aggregate shocks, such that the fraction of impatient depositors is always $\overline{\lambda}$, then the first-order conditions and binding constraints give the well-known first best allocation, denoted with asterisks, as implicitly defined by

$$u'(c_1^*) = ru'(c_2^*) \tag{7}$$

$$\overline{\lambda}c_1^* = 1-\alpha^* \tag{8}$$

$$\left(1-\overline{\lambda}\right)c_2^* = \alpha^* r \tag{9}$$

$$\beta^* = 0. \tag{10}$$

Equation (7) shows that the ratio of marginal utilities between dates 1 and 2 is equal to the marginal return on investment r.

If there are aggregate shocks, the planner's problem is identical to the problem described in Allen, Carletti, and Gale (2008), who show that there exists a unique solution to this problem. Intuitively, the planner's allocation with aggregate shocks is constructed as follows. The planner stores just enough goods to provide consumption to all impatient agents in the state with many impatient agents, $\alpha = H$. This implicitly defines c_1^*. In this state, patient agents consume only goods invested in the long-term technology. In the state with few impatient agents, $\alpha = L$, the planner stores $(\overline{\lambda}_H - \overline{\lambda}_L)c_1^*$ goods in excess of what is needed for impatient agents. These goods are stored between dates 1 and 2 and given to patient agents.

4 Equilibrium without aggregate shocks

To simplify the exposition, we first consider the case where there are no aggregate shocks. Next, we consider the case with both idiosyncratic and aggregate shocks. We will show that the central bank uses different tools to respond to each shock.

We assume that the fraction of impatient depositors is always $\overline{\lambda}$. Consider the optimization problem of a bank of type j given by (4).

Lemma 1. *First-order conditions with respect to c_1 and α are, respectively,*

$$u'(c_1) = E[\frac{\lambda^{ij}}{\overline{\lambda}}\iota^i u'(c_2^{ij})] \tag{11}$$

$$E[\iota^i u'(c_2^{ij})] = rE[u'(c_2^{ij})]. \tag{12}$$

Proof. The Lagrange multiplier for constraint (5) is θ_β^{ij}. The first-order condition with respect to β^{ij} is

$$\begin{aligned} \tfrac{1}{2}\rho u'(c_2^{1j})(1-\iota^1) &\leq \theta^{1j} \text{ for } j \in \mathcal{J} \qquad (= \text{ if } \beta^{1j} > 0) \\ (1-\rho)u'(c_2^{0j})(1-\iota^0) &\leq \theta^{0j} \text{ for } j \in \mathcal{J} \qquad (= \text{ if } \beta^{0j} > 0), \end{aligned}$$

which for $\iota^i > 1$ does not bind and implies $\beta^{ij} = 0$, and for $\iota^i = 1$ implies $\beta^{ij} = 0$ since banks are indifferent between storing and lending goods. Complementary slackness for constraint (5) implies $\theta_\beta^{ij} = 0$. First-order conditions (11) and (12) follow. ■

Equation (11) is the Euler equation and determines the investment level α given ι^i for $i \in \mathcal{I}$. Equation (12) is a no-arbitrage pricing condition for the rate ι^i, which states that the expected marginal utility-weighted returns on storage and investment must be equal. The return on investment between dates 0 and 2 is r. The return on storage between dates 0 and 2 is the market rate ι^i. Banks can store goods at date 0, lend them at date 1, and will receive ι^i at date 2. The rates ι^1 and ι^0 are determined in equilibrium to make banks indifferent to holding goods and assets at date 0.

The clearing condition for the interbank market is

$$f^{ih} = -f^{il} \text{ for } i \in \mathcal{I},$$

which, together with the bank's budget constraints (1) and (2), determine $c_1^j(\alpha)$ and $f^{ij}(\alpha)$ as functions of α:

$$\begin{aligned} c_1(\alpha) &= \frac{1-\alpha}{\overline{\lambda}} \\ f^{ij}(\alpha) &= (1-\alpha)(\frac{\lambda^{ij}}{\overline{\lambda}} - 1). \end{aligned}$$

Finding the market equilibrium is reduced to solving the two first-order conditions, equations (11) and (12), in three unknowns, α, ι^1, and ι^0. Since no goods are stored between dates 1 and 2 for idiosyncratic state $i = 0, 1$, average consumption by patient consumers equals $\frac{\alpha r}{1-\overline{\lambda}}$ in each idiosyncratic state. For simplicity of notation, we can write the average consumption by patient consumers in both idiosyncratic states $i = 0, 1$ as

$$c_2^{0j}(\alpha) = \frac{\alpha r}{1-\overline{\lambda}}.$$

We can also write

$$c_2^{0j}(\alpha) = \frac{(1-\lambda^{1h})c_2^{1h} + (1-\lambda^{1l})c_2^{1l}}{1-\overline{\lambda}};$$

the right-hand side of the equation gives an alternate expression for average consumption by patient depositors in state $i = 1$.

4.1 Single state: $\rho = 0, 1$

We start by finding solutions to the special cases of $\rho = 0, 1$. These are particularly interesting benchmarks, as the first one ($\rho = 0$) corresponds to the standard framework of Diamond-Dybvig, while the second one ($\rho = 1$) corresponds to the case studied by

Bhattacharya and Gale (1987). These boundary cases will then help us to solve the general model $\rho \in [0,1]$. There is certainty about the single state of the world i at date 1. First-order conditions (11) and (12) can be written more explicitly as

$$\begin{aligned} & \rho[\tfrac{1}{2}u'(c_2^{1h}) + \tfrac{1}{2}u'(c_2^{1l})]\iota^1 + (1-\rho)u'(c_2^{0j})\iota^0 \\ = \;& \rho[\tfrac{1}{2}u'(c_2^{1h}) + \tfrac{1}{2}u'(c_2^{1l})]r + (1-\rho)u'(c_2^{0j})r \end{aligned} \tag{13}$$

$$u'(c_1) = \rho[\tfrac{\lambda^{1h}}{2\overline{\lambda}}u'(c_2^{1h}) + \tfrac{\lambda^{1l}}{2\overline{\lambda}}u'(c_2^{1l})]\iota^1 + (1-\rho)u'(c_2^{0j})\iota^0. \tag{14}$$

Equations (13) and (14) imply that for $\rho = 0$, the value of ι^1 is indeterminate, and for $\rho = 1$, the value of ι^0 is indeterminate. In either case, we will show that there is an equilibrium with unique values for the allocation c_1, c_2^{ij}, and α. The indeterminate variable is of no consequence for the allocation. The allocation is determined by the two first-order equations, in the two unknowns α and ι^0 (for $\rho = 0$) or ι^1 (for $\rho = 1$). The first-order condition with respect to α, equation (13), shows that the interbank lending rate equals the return on assets: $\iota^0 = r$ (for $\rho = 0$) or $\iota^1 = r$ (for $\rho = 1$). With a single state of the world, the interbank lending rate must equal the return on assets.

In the case of no shock with $\rho = 0$, the banks' budget constraints imply that in equilibrium no interbank lending occurs, $f^{0j} = 0$ for $j \in \mathcal{J}$. The interbank lending rate ι^0 is the lending rate at which each bank's excess demand is zero. The Euler equation (14) for bank j is equivalent to equation (7) for the planner. Banks choose the optimal α^* and provide the first best allocation c_1^* and c_2^*, which are illustrated in Figure 1.

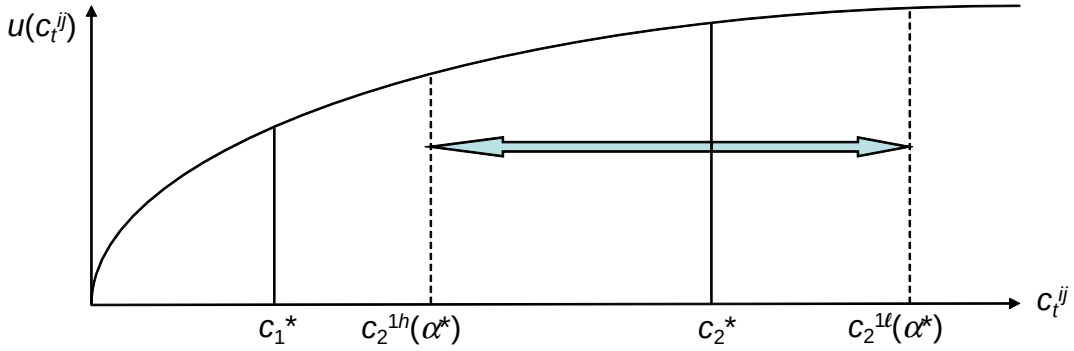

Figure 1

Banks provide liquidity at date 1 to impatient consumers by paying $c_1^* > 1$. This can be accomplished only by paying $c_2^* < r$ on withdrawals to patient consumers at date 2.

The key for the bank being able to provide liquidity insurance to impatient consumers is that the bank can pay only an implicit date 1 to date 2 intertemporal return on deposits of $\frac{c_2^*}{c_1^*}$, which is less than the return on assets r. This contract is optimal because the ratio of intertemporal marginal utility equals the marginal return on assets, $\frac{u'(c_2^*)}{u'(c_1^*)} = r$.

Proposition 1. *For $\rho = 0$, there exists a rational expectations equilibrium characterized by $\iota^0 = r$ that has a unique first best allocation c_1^*, c_2^*, α^*.*

Proof. For $\rho = 0$, equation (13) implies $\iota^0 = r$. Equation (14) simplifies to $u'(c_1) = u'(c_2^{0j})r$, and the bank's budget constraints bind and simplify to $c_1 = \frac{1-\alpha}{\lambda}$, $c_2^{0j} = \frac{\alpha r}{1-\lambda}$. These results are equivalent to the planner's results in equations (7) through (9), implying there is a unique equilibrium, where $c_1 = c_1^*$, $c_2^{0j} = c_2^*$, and $\alpha = \alpha^*$. ■

In the case of a certain shock with $\rho = 1$, there is interbank lending. The banks' budget constraints imply that in equilibrium $f^{1h} = \varepsilon c_1$ and $f^{1l} = -\varepsilon c_1$. First, consider the outcome at date 1 holding fixed $\alpha = \alpha^*$. With $\iota^1 = r$, we will show that the patient consumers do not have optimal consumption: $c_2^{1h}(\alpha^*) < c_2^* < c_2^{1l}(\alpha^*)$. The deviation from optimality is illustrated by the arrows in Figure 1. A bank of type h has to borrow at date 1 at the rate $\iota^1 = r$, a rate that is higher than the optimal rate between dates 1 and 2 paid to patient depositors of $\frac{c_2^*}{c_1^*}$. Late consumers face risk to their consumption conditional on being a patient type.

Second, consider the determination of α. We will show that the equilibrium investment is $\alpha > \alpha^*$. Compared to the first best, banks store fewer liquid goods at date 0 and pay lower c_1 at date 1 in order to hold more assets that provide banks greater self-insurance liquidity available at date 2 to pay to patient consumers. This is justified because, for the original allocation, under risk aversion, patient consumers have a lower expected utility. To make up for this lower utility of patient consumers, a redistribution of ex-ante utilities detrimental to impatient consumers has to take place. The difference of equilibrium consumption compared to consumption for a fixed $\alpha = \alpha^*$ is demonstrated by the arrows in Figure 2. The result is $c_1 < c_1^*$, $c_2^{0j} > c_2^*$, $c_2^{1h} > c_2^{1h}(\alpha^*)$, and $c_2^{1l} > c_2^{1l}(\alpha^*)$. For any $\varepsilon > 0$ shock, banks do not provide the optimal allocation.

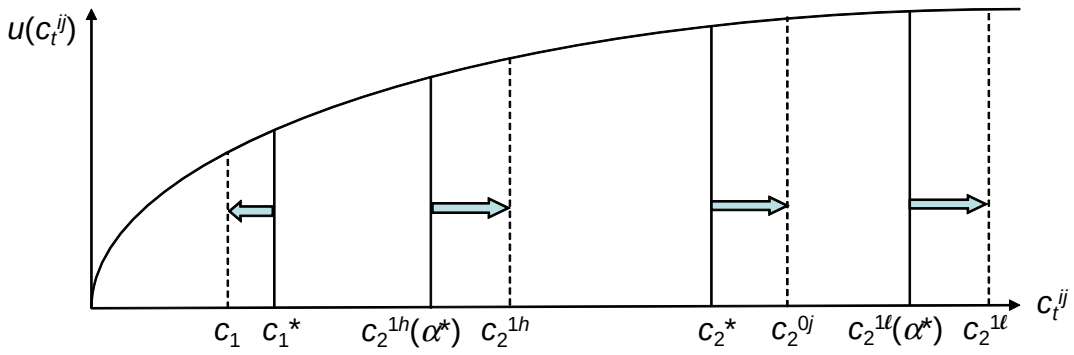

Figure 2

Proposition 2. *For $\rho = 1$, there exists a rational expectations equilibrium characterized by $\iota^1 = r$ that has a unique suboptimal allocation*

$$\begin{aligned} c_1 &< c_1^* \\ c_2^{1h} &< c_2^* < c_2^{1l} \\ \alpha &> \alpha^*. \end{aligned}$$

Proof. For $\rho = 1$, equation (13) implies $\iota^1 = r$. By equation (3), $c_2^{1l} > c_2^{1h}$. From the bank's budget constraints and market clearing,

$$\frac{1-\overline{\lambda}-\varepsilon}{2(1-\overline{\lambda})}c_2^{1h} + \frac{1-\overline{\lambda}+\varepsilon}{2(1-\overline{\lambda})}c_2^{1l} = \frac{\alpha r}{1-\overline{\lambda}} = c_2^{0j},$$

which implies $\frac{1}{2}c_2^{1h} + \frac{1}{2}c_2^{1l} < c_2^{0j}$, since $c_2^{1l} > c_2^{1h}$. Because $u(\cdot)$ is concave, $\frac{1}{2}u'(c_2^{1h}) + \frac{1}{2}u'(c_2^{1l}) > u'(c_2^{0j})$. Further, $\frac{\lambda^{1h}}{2\overline{\lambda}}u'(c_2^{1h}) + \frac{\lambda^{1l}}{2\overline{\lambda}}u'(c_2^{1l}) > u'(c_2^{0j})$ since $\lambda^{1h} > \lambda^{1l}$, $\frac{\lambda^{1h}}{2\overline{\lambda}} + \frac{\lambda^{1l}}{2\overline{\lambda}} = 1$ and $c_2^{1h} < c_2^{1l}$. Thus,

$$\begin{aligned} u'(c_1(\alpha^*)) &= ru'(c_2^{0j}(\alpha^*)) \\ &< r[\frac{\lambda^{1h}}{2\overline{\lambda}}u'(c_2^{1h}(\alpha^*)) + \frac{\lambda^{1l}}{2\overline{\lambda}}u'(c_2^{1l}(\alpha^*))]. \end{aligned}$$

Since $u'(c_1(\alpha))$ is increasing in α and $u'(c_2^{1j}(\alpha))$ for $j \in \mathcal{J}$ is decreasing in α, the Euler equation implies that, in equilibrium, $\alpha > \alpha^*$. Hence, $c_1 = \frac{1-\alpha}{\overline{\lambda}} < c_1^*$, $c_2^{1l} > c_2^{0j} = \frac{\alpha r}{1-\overline{\lambda}} > c_2^*$ and $c_2^{1h} < c_2^*$. ■

Notice that for $\rho = 1$, the difference between our approach and that of Bhattacharya and Gale (1987) is that in our framework the market cannot impose any restriction on the size of the trades. This forces the interbank market to equal r and creates an inefficiency. Their mechanism design approach yields a second best allocation that achieves higher

welfare, but in that case, the market cannot be anonymous anymore, as the size of the trade has to be observed and enforced.

4.2 General shock: $\rho \in [0,1]$

We now apply our results of the special cases of $\rho = 0, 1$ to examine the general case of $\rho \in [0,1]$. We will show that there are multiple rational expectations equilibria with different real allocations of c_1, c_2^{ij}, and α.

There are two possible idiosyncratic states of the world at date 1: $i = 1, 2$. An equilibrium is determined by two equations, first order condition (13) and (14), in three unknowns, α, ι^1, and ι^0. This will be a key difference with respect to the benchmark cases, as now the bank is facing a distribution of probabilities over two interest rates, while in the two previous cases either the interest rate was irrelevant (and indeterminate) or it was uniquely determined by the long-run technology.

The bank's budget constraints imply that, in the state of no shock with $i = 0$, no interbank lending occurs, $f^{ja} = f^{jb} = 0$, and

$$c_2^{0j} = \frac{\alpha r}{1 - \overline{\lambda}}, \tag{15}$$

as in the case of $\rho = 0$. In the state of a positive shock with $\varepsilon > 0$, there is interbank lending with $f^{1h} = \varepsilon c_1$, $f^{1l} = -\varepsilon c_1$,

$$c_2^{ij} = \frac{\alpha r - (\lambda^{ij} - \overline{\lambda}) c_1 \iota^i}{1 - \lambda^{ij}}. \tag{16}$$

First, we show that there exists a suboptimal rational expectations equilibrium with $\iota^1 = \iota^0 = r$. Consider $\iota^1 = r$. Equation (13) implies $\iota^0 = r$. Equation (14) is a single equation with a single unknown α, which is determined. Equation (14) implies that $\alpha(\rho)$ is an implicit function of ρ. Likewise, $c_2^{0j}(\rho)$, $c_2^{1h}(\rho)$, and $c_2^{1l}(\rho)$ are implicit functions of ρ. We can use the cases of $\rho = 0$ and $\rho = 1$ to provide bounds for the general case of $\rho \in [0,1]$. The equilibrium $c_1(\rho)$ and $c_2^{ij}(\rho)$ for $i \in \mathcal{I}$, $j \in \mathcal{J}$, written as functions of ρ, are displayed in Figure 3. This figure shows that $c_1(\rho)$ is decreasing in ρ while $c_2^{ij}(\rho)$ is increasing in ρ:

$$\begin{aligned} c_2^{ij}(0) &\leq c_2^{ij}(\rho) \leq c_2^{ij}(1) \qquad \text{for } \rho \in [0,1],\ i \in \mathcal{I},\ j \in \mathcal{J} \\ c_1(1) &\leq c_1(\rho) \leq c_1(0) \qquad \text{for } \rho \in [0,1]. \end{aligned}$$

In addition,

$$\begin{aligned} c_2^{0j}(\rho=0) &= c_2^* && \text{for } j \in \mathcal{J} \\ c_1(\rho=0) &= c_1^* && \\ c_2^{1j}(\rho=0) &= c_2^{1j}(\alpha=\alpha^*) && \text{for } j \in \mathcal{J}. \end{aligned}$$

With interbank rates equal to r in all states, there is inefficient risk-sharing among patient consumers. To compensate, there is inefficient liquidity provided to impatient consumers.

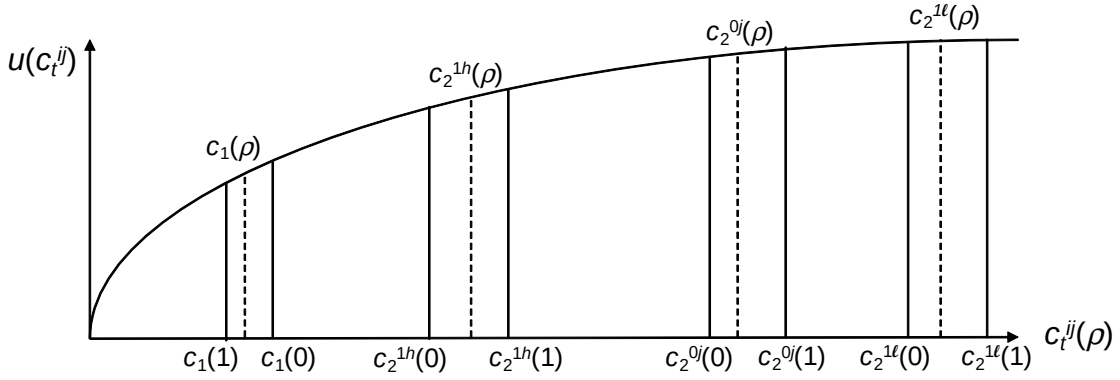

Figure 3

Second, we show for $\rho < 1$ that there also exists a first best rational expectations equilibrium with

$$\iota^1 = \iota^{1^*} \equiv \frac{c_2^{0j}}{c_1} \qquad \text{for } j \in \mathcal{J}. \tag{17}$$

To show this, first we substitute for c_2^{0j} from equation (15) into equation (17) and for ι^1 from equation (17) into equation (16) and simplify, which gives

$$c_2^{1h} = c_2^{1l} = c_2^{0j} = \frac{\alpha r}{1-\overline{\lambda}}.$$

With ι^1 equal to the intertemporal return on deposits between dates 1 and 2, there is optimal ex-post risk-sharing of the goods that are available at date 2 through interbank lending at the low rate at date 1. Substituting for ι^1 and c_2^{1j} into equation (13) and rearranging gives

$$\iota^0 = r + \frac{\rho(r - \frac{c_2^{0j}}{c_1})}{1-\rho}. \tag{18}$$

Substituting for c_2^{1j}, ι^1, and ι^0 into equation (14) and rearranging gives $u'(c_1) = r'u'(c_2^{0j})$. This is the planner's condition, and implies $\alpha = \alpha^*$, $c_1 = c_1^*$, and $c_2^{0j} = c_2^*$,

a first best allocation. To interpret, substituting these equilibrium values into equations (17) and (18) and simplifying shows that

$$\begin{aligned} \iota^1 &= \iota^{1^*} = \frac{c_2^*}{c_1^*} < r \\ \iota^0 &= \iota^{0^*} \equiv r + \frac{\rho(r - \frac{c_2^*}{c_1^*})}{1-\rho} > r. \end{aligned} \tag{19}$$

With ι^0 greater than r during the no-shock state, there is no ex-post inefficiency because there is no need for interbank lending. With ι^1 less than r for the shock state, there is no ex-post inefficiency with interbank lending because the rate is at the low optimal rate. The following result shows that the expected interbank rate is equal to the return on assets. This result is based on the first-order condition with respect to α, which requires banks to be willing to hold both storage and investment at date 0.

Proposition 3. *The expected interbank rate is* $E[\iota^i] = r$.

Proof. $E[\iota^i] = \rho\iota^1 + (1-\rho)\iota^0$. Substituting for ι^1 and ι^0 from (17) and (19) and simplifying, $E[\iota^i] = r$. ■

Since there is no risk to patient consumers, banks hold optimal α^*. Figure 4 illustrates the distinction of this first best equilibrium (with ι^{1^*}, ι^{0^*}) from the suboptimal equilibrium (with $\iota^1 = \iota^0 = r$). Arrows indicate that in contrast with the suboptimal $\iota^i = r$ equilibrium, in the $\iota^1 = \frac{c_2^*}{c_1^*}$ equilibrium we find the first best outcome that $c_2^{ij}(\rho) = c_2^*$ and $c_1(\rho) = c_1^*$ for all $i \in \mathcal{I}$, $j \in \mathcal{J}$, and $\rho < 1$.

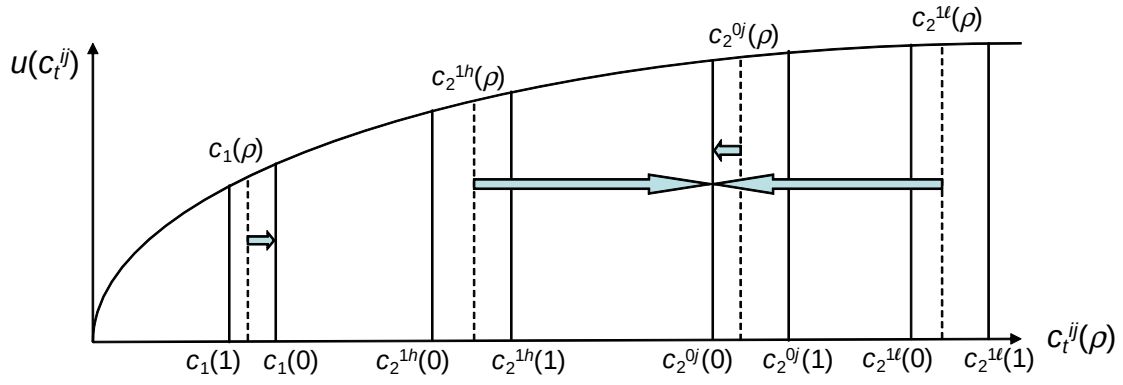

Figure 4

For $\rho = 1$, $\iota^1 = \frac{c_2^{0j}}{c_1}$ would imply ι^0 is not finite and equations (13) and (14) are not well specified. Therefore, we rule out $\iota^1 = \frac{c_2^{0j}}{c_1}$ as an equilibrium value for $\rho = 1$. As in

the case of $\rho = 1$ above, there are multiple equilibria since ι^1 is indeterminate, but the allocation α, c_1, c_2^{0j} is unique and not first best. The following proposition summarizes the results we have just shown.

Proposition 4. *For $\rho \in (0,1)$, there exist multiple rational expectations equilibria with different allocations. There exists a suboptimal rational expectations equilibrium with*

$$\begin{aligned} \iota^1 &= \iota^0 = r \\ \alpha &> \alpha^* \\ c_1 &< c_1^* \\ c_2^{0j} &> c_2^* \\ c_2^{1h} &< c_2^* < c_2^{1l}, \end{aligned}$$

and there exists a first best rational expectations equilibrium with

$$\begin{aligned} \iota^1 &= \frac{c_2^*}{c_1^*} < r \\ \iota^0 &= \iota^{0*} > r \\ \alpha &= \alpha^* \\ c_1 &= c_1^* \\ c_2^{ij} &= c_2^* \qquad \textit{for } i \in \mathcal{I},\ j \in \mathcal{J}. \end{aligned}$$

Our result is novel in showing that because there are multiple idiosyncratic liquidity states i at date 1, there exist multiple rational expectations equilibria from the perspective of date 0. Allen and Gale (2004) show that there exist sunspot ex-post equilibria in this type of model. From the ex-post perspective of date 1 only, an indeterminate continuum of ι^i is consistent with ex-post individual rationality for banks lending in the interbank market. We show that there is a family of ι^1, ι^0 at date 1, each pair of which can be anticipated and support a different rational expectations equilibrium. Within a rational expectations equilibrium, ι^1 and ι^0 do not need to be equal. The results from this section generalize in a straightforward way to the case of N states, as shown in Appendix A.

4.3 The role of the central bank's policy

The result of multiple Pareto-ranked equilibria in our model suggests a role for an institution that can select the best equilibrium. Since equilibria can be distinguished by

the interest rate in the interbank market, a central bank is the natural candidate for this role. We think of the interest rate ι^i at which banks lend in the interbank market as the unsecured interest rate that many central banks target for monetary policy. In the U.S. the Federal Reserve targets the overnight interest rate, also known as the federal funds rate.

The central bank can set the interbank rate at a low level $\iota^1 = \frac{c_2^*}{c_1^*}$ when the idiosyncratic shock state $i = 1$ is realized. This policy has distributional effects since lowering the interest rate, from, for example, r to $\frac{c_2^*}{c_1^*}$, increases the consumption of patient depositors in banks of type h and reduces the consumption of patient depositors in banks of type l. By equalizing the consumption of patient depositors in both kinds of banks, this policy achieves optimal risk-sharing. This allows banks to reduce the expected consumption of patient depositors, since they no longer need to be compensated for consumption risk. Banks can hold more liquid goods, which allows them to offer better insurance to depositors against their preference shock. Extra high rates of $\iota^0 = \iota^{0^*} > r$ are required when the state $i = 0$ with no idiosyncratic shock occurs, such that expected rates equal the return on assets, $E[\iota^i] = r$, and banks are indifferent between holding goods and assets at date 0.

A central bank can achieve the desired interest rate by promising to borrow or lend goods at that rate. This policy resembles a corridor system of monetary policy, with a corridor of zero width. We provide a formal expansion of the model that shows how the central bank can actively select and enforce its choice of interbank rates in Appendix B. Bank deposit contracts are expressed in nominal terms and fiat money is borrowed and lent in the interbank market, along the lines of Skeie (2008) and Martin (2006). In this richer setting, we show explicitly that the central bank can offer to borrow and lend unlimited amounts of fiat money at its nominal policy rate contingent on the state i at date 1. This forces banks to trade at this rate in the interbank market, and the central bank does zero borrowing and lending in equilibrium.

5 Equilibrium with aggregate shocks

The aggregate fraction of depositors in the economy can take two values: $\overline{\lambda}_L$, with probability $\pi \in [0, 1]$, and $\overline{\lambda}_H$, with probability $1 - \pi$, $\overline{\lambda}_H > \overline{\lambda}_L$. We assume that the central bank can tax the endowment of agents at date 0, store these goods, and return the taxes at

date 1 or at date 2. We denote these transfers, which can be conditional on the aggregate shock, τ_0, τ_{1a}, τ_{2a}, $a \in \mathcal{A}$, respectively.

Banks aim to maximize

$$\begin{aligned} E[U] &= \left[\pi\overline{\lambda}_H + (1-\pi)\overline{\lambda}_L\right] u(c_1) \\ &\quad +(1-\rho)\left[\pi(1-\overline{\lambda}_H)u(c_{2H}^0) + (1-\pi)(1-\overline{\lambda}_L)u(c_{2L}^0)\right] \\ &\quad +\rho\frac{\pi}{2}\left[(1-\lambda_H^{1h})u(c_{2H}^{1h}) + (1-\lambda_H^{1l})u(c_{2H}^{1l})\right] \\ &\quad +\rho\frac{1-\pi}{2}\left[(1-\lambda_L^{1h})u(c_{2L}^{1h}) + (1-\lambda_L^{1l})u(c_{2L}^{1l})\right], \end{aligned}$$

subject to

$$\begin{aligned} \lambda_a^{ij} c_1 &= 1 - \tau_0 - \alpha - \beta_a^{ij} + f_a^{ij} + \tau_{1a}, \text{ for } a \in \mathcal{A},\ i \in \mathcal{I},\ j \in \mathcal{J} \\ (1-\lambda_a^{ij}) c_{a2}^{ij} &= \alpha r + \beta_a^{ij} - f_a^{ij}\iota_a^i + \tau_{2a}, \text{ for } a \in \mathcal{A},\ i \in \mathcal{I},\ j \in \mathcal{J}, \end{aligned}$$

where c_{2a}^{ij} denotes consumption at date 2 for an impatient depositor of bank $j \in \mathcal{J}$ in idiosyncratic state $i \in \mathcal{I}$ and aggregate state $a \in \mathcal{A}$.

The first-order conditions take the same form as in the case without aggregate risk and become:

$$u'(c_1) = E\left[\frac{\lambda_a^{ij}}{\pi\overline{\lambda}_H + (1-\pi)\overline{\lambda}_L}\iota_a^i u'(c_{a2}^{ij})\right] \tag{20}$$

$$E[\iota_a^i u'(c_{a2}^{ij})] = rE[u'(c_{a2}^{ij})]. \tag{21}$$

Assume that the amount of stored goods that the central bank taxes is $\tau_0 = (\overline{\lambda}_H - \overline{\lambda}_L)c_1$. Consider the economy in the case where $i = 0$: If there are many impatient depositors, the banks will not have enough stored goods for their impatient depositors. However, the central bank can return the taxes at date 1, setting $\tau_{1H} = (\overline{\lambda}_H - \overline{\lambda}_L)c_1$ (and $\tau_{2H} = 0$), so that banks have enough stored goods. In that case, banks have just enough goods for their impatient depositors. There is no activity in the interbank market, and the interbank market rate is indeterminate. If there are few impatient depositors and the central bank sets $\tau_{1L} = 0$ (with $\tau_{2L} = (\overline{\lambda}_H - \overline{\lambda}_L)c_1$), then banks have just enough goods for their impatient depositors at date 1. Again, there is no activity in the interbank market, and the interbank market rate is indeterminate.

Now consider the economy with idiosyncratic shocks, $i = 1$. If there are many impatient depositors, the banks do not have enough stored goods, on aggregate, for their impatient

depositors. However, as in the previous case, the central bank can return the taxes at date 1, setting $\tau_{1H} = (\overline{\lambda}_H - \overline{\lambda}_L)c_1$ (and $\tau_{2H} = 0$), so that banks have enough stored goods on aggregate. The interbank market interest rate is indeterminate, since the supply and demand of stored goods are inelastic, so the central bank can choose the rate to be $\iota^0 = \frac{c_2^*}{c_1^*}$. If there are few impatient depositors, the central bank sets $\tau_{1L} = 0$ (with $\tau_{2L} = (\overline{\lambda}_H - \overline{\lambda}_L)c_1$) and $\iota^0 = \frac{c_2^*}{c_1^*}$.

In the cases where $i = 0$, no matter what the aggregate shock is, the interbank market rate can be chosen to make sure that equation (21) holds. With such interbank market rates, banks will choose the optimal investment. Indeed, since equation (21) holds, banks are willing to invest in both storage and the long-term technology. In states where there is no idiosyncratic shock, there is no interbank market lending, so any deviation from the optimal investment carries a cost. In states where there is an idiosyncratic shock, the rate on the interbank market is such that the expected utility of a bank's depositors cannot be higher than under the planner's allocation, so there is no benefit from deviating from the optimal investment in these states.

In our model, the central bank uses different tools to deal with aggregate and idiosyncratic shocks. When an aggregate shock occurs, the central bank needs to inject liquidity in the form of stored goods. In contrast, when an idiosyncratic shock occurs, the central bank needs to lower interest rates. Note that these two policy tools do not interact. The central bank should apply both tools simultaneously whenever an aggregate shock and an idiosyncratic shock occur simultaneously.

During the recent crisis, certain central banks have been using tools that some believe are more appropriately thought of as part of fiscal policy. This is consistent with our model in that the central bank policy of taxing and redistributing goods in the case of aggregate shocks is similar to fiscal policy. The model does not imply that the central bank should be the preferred institution to implement this kind of policy. For example, we could assume that different institutions are in charge of i) setting the interbank rate, and ii) choosing τ_0, τ_{1a}, τ_{2a}, $a \in \mathcal{A}$. Regardless of the choice of institutions, our model suggests that tools resembling fiscal policy may be needed to address aggregate liquidity shocks.

6 Financial fragility

The main role of the central bank's policy response to idiosyncratic shocks is to improve risk-sharing among the banks' patient depositors. In this section, we illustrate the importance of this policy by showing that it can help prevent financial fragility. In the state where $i = 1$, patient depositors of banks with many impatient agents will consume less than patient depositors of other banks if the central bank sets the interest rate higher than $\frac{c_2^*}{c_1^*}$, the optimal return on deposits between dates 1 and 2. If ε is large, it may be the case that the consumption of patient depositors of banks with many impatient agents would be lower if they withdraw at date 2 than if they withdraw at date 1, which would trigger a bank run.

This argument can be presented in several ways. One way is to find the equilibrium allocation assuming that the central bank does not follow the optimal policy and show that, in equilibrium, bank runs would occur at institutions that have many impatient depositors. An alternative approach is to consider an equilibrium assuming that the central bank promises to follow the optimal policy and show that, if the central bank makes an unexpected mistake, a bank run occurs. We consider each approach, starting with the latter.

6.1 Central bank makes unexpected mistake

To simplify the exposition, we assume that the probability of an aggregate liquidity shock is zero, such that the fraction of impatient depositors is always $\overline{\lambda}$. At date 0, banks expect the central bank to follow the optimal policy. However, suppose that, unexpectedly, the central bank chooses to deviate from the optimal interest rate policy and sets an interest rate $\iota^1 = r > \frac{c_2^*}{c_1^*}$ in the state where $i = 1$. In this case, the consumption, c_2^{1h}, of patient depositors in banks with many impatient agents is

$$c_2^{1h} = \frac{\alpha^* r - \varepsilon c_1^* r}{1 - \overline{\lambda} - \varepsilon} = \frac{r}{1 - \overline{\lambda} - \varepsilon} \left[\alpha^* - \varepsilon \frac{1 - \alpha^*}{\overline{\lambda}} \right],$$

since $c_1^* = (1 - \alpha^*)/\overline{\lambda}$. If we assume that the utility function is of the form

$$u(c) = \frac{c^{1-\sigma}}{1 - \sigma}, \quad \sigma > 1,$$

then we can rewrite the expression for c_2^{1h} as

$$c_2^{1h} = \frac{\alpha^* r}{1-\overline{\lambda}} \left[\frac{1-\overline{\lambda}-\varepsilon r^{\frac{\sigma-1}{\sigma}}}{1-\overline{\lambda}-\varepsilon} \right].$$

Recall that $\varepsilon \leq \min\{\overline{\lambda}, 1-\overline{\lambda}\}$. If $\overline{\lambda}$ is very small, then ε must also be very small and the term in brackets will be close to 1. This implies that c_2^{1h} will be close to c_2^* and no bank run can occur since $c_2^* > c_1^*$. In contrast, if $\overline{\lambda} \geq 1/2$, then the term in brackets can be made arbitrarily close to zero, since $r > 1$ so that c_2^{1h} will be close to zero. In such cases, bank runs can occur.

Consider the following example: $\overline{\lambda} = 1-\overline{\lambda} = 1/2$, $r = 1.5$, and $\sigma = 2$. For such parameters, we have $\alpha^* \approx 0.4495$, $c_1^* \approx 1.101$, and $c_2^* \approx 1.3485$. Now assume that $\varepsilon = 0.3$; then $c_2^{1h} \approx 0.8939 < c_1^*$, and a bank run would result.

6.2 Runs in equilibrium

Consider the equilibrium allocation if banks anticipate that the interbank market interest rate will be $\iota^1 = \iota^0 = r$. By continuity, this allocation converges to the optimal allocation as $\rho \to 0$. We have already seen that at the optimal level α^*, bank runs can occur if i is sufficiently large and $\iota^1 = r$. Now since bank runs are anticipated, banks could choose a "run preventing" deposit contract, as suggested by Cooper and Ross (1998). However, following the argument in that paper, banks will not choose a run-preventing deposit contract if the probability of a bank run is sufficiently small. So for ρ sufficiently close to zero, bank runs will occur in equilibrium.

7 Liquidation of the long-term technology

We extend the model to allow for liquidation of the investment at date 1. Again, to simplify the exposition, we assume that the fraction of impatient depositors is always $\overline{\lambda}$. We show that this restricts possible real interbank rates and may preclude the first best equilibrium.

At date 1, bank j liquidates γ^{ij} of the investment for a salvage rate of return s at date 1 and no further return at date 2. The bank budget constraints (1) and (2) are replaced

by

$$\begin{aligned} \lambda^{ij} c_1 &= 1 - \alpha - \beta^{ij} + \gamma^{ij} s + f^{ij} \qquad \text{for } i \in \mathcal{I},\ j \in \mathcal{J} \\ (1 - \lambda^{ij}) c_2^{ij} &= (\alpha - \gamma^{ij}) r + \beta^{ij} - f^{ij} \iota^i \qquad \text{for } i \in \mathcal{I},\ j \in \mathcal{J}, \end{aligned}$$

and the bank optimization (4) is replaced by

$$\begin{aligned} \max_{\alpha, c_1, \{\beta^{ij}, \gamma^{ij}\}_{i \in \mathcal{I}, j \in \mathcal{J}}} \quad & E[U] \\ \text{s.t.} \quad & \beta^{ij} \leq 1 - \alpha \qquad \text{for } i \in \mathcal{I},\ j \in \mathcal{J} \\ & \gamma^{ij} \leq \alpha \qquad \text{for } i \in \mathcal{I},\ j \in \mathcal{J}. \end{aligned} \tag{22}$$

The first-order condition with respect to γ^{ij} is

$$\tfrac{1}{2} \rho u'(c_2^{1j})(\iota^1 s - r) \leq \theta_\gamma^{1j} \ \text{ for } j \in \mathcal{J} \qquad (= \text{ if } \gamma^{1j} > 0) \tag{23}$$

$$(1 - \rho) u'(c_2^{0j})(\iota^0 s - r) \leq \theta_\gamma^{0j} \ \text{ for } j \in \mathcal{J} \qquad (= \text{ if } \gamma^{0j} > 0), \tag{24}$$

where θ_γ^{ij} is the Lagrange multiplier for constraint (22). Without loss of generality, we assume that no bank j liquidates all investment in state i unless all banks do. Because the interbank market is ex-post efficient, the equilibrium and allocation depend solely on the aggregate amount of liquidation, not the distribution of liquidation among banks. If there is complete liquidation of investment, then clearly the allocation is not first best.

Consider an equilibrium in which there is no complete liquidation of investment. Complementary slackness for constraint (22) implies $\theta_\gamma^{ij} = 0$. Conditions (23) and (24) can be written as

$$\iota^i \leq \frac{r}{s} \qquad \text{for all } i \in \mathcal{I},$$

which gives a restriction on the equilibrium interest rate in state i. The intuition for this result is simply that too high an interest rate ι^i would make it profitable for banks to liquidate their assets in order to lend in the interbank market.

If there is liquidation by any bank j in any state $i \in \mathcal{I}$, the equilibrium is not first best. Alternatively, if $\iota^{0^*} > \frac{r}{s}$, then the equilibrium cannot be first best. The interest cannot be high enough in the $i = 0$ state. At an interest rate of $\iota^0 > \frac{r}{s}$, all banks would liquidate investment and lend it on the interbank market, and no banks would borrow, which cannot be an equilibrium.

It is interesting to emphasize that as s stands for salvage value of the investment, it can be interpreted as the liquidity of a market for the long-run technology. From that

perspective, our result states that the higher the liquidity of the market for the long-term technology, the lower the ex-ante efficiency of the banking system. Our result is surprising in the context of monetary policy, but it is quite natural in the context of Diamond-Dybvig models, where the trading of deposits destroys the liquidity insurance function of banks.

8 Conclusion

This paper examines the ex-ante choice of bank liquidity and the ex-post reallocation of bank liquidity through the interbank market after random idiosyncratic and aggregate liquidity shocks. We show that the central bank can achieve the full-information first best allocation with two different tools, one for each type of shock. The central bank should address idiosyncratic liquidity shocks by lowering the interbank market rate, and it should respond to aggregate liquidity shocks by injecting liquid assets into the banking system. We also show that a failure to follow the optimal policy can lead to financial fragility.

In our model, a high expected interest rate is necessary, ex ante, to provide incentives for banks to hold both liquid and illiquid assets. Ex post, the level of the interest rate will determine how efficiently liquidity is shared in the interbank market. If an idiosyncratic liquidity shock occurs, banks with excess liquidity will want to lend it in the market, while banks with a shortage of liquidity will want to borrow. If the interbank market rate is high when such a shock occurs, patient depositors of different banks will face unequal consumption. Patient depositors in banks that lend at a high rate will consume more than those in banks that borrow at that rate. This consumption inequality is inefficient and can also lead to bank runs.

With state-dependant interbank market rates, we show that there are multiple rational expectations equilibria, which are Pareto-ranked from an ex-ante point of view. One of these equilibria achieves the optimal allocation. In the optimal equilibrium, the interbank market interest rate is low when an idiosyncratic shock occurs. It is set so that consumption inequality between patient depositors of different banks is eliminated. To maintain a sufficiently high ex-ante expected interest rate, the interbank rate is required to be particularly high whenever idiosyncratic shocks do not occur. A central bank is a natural candidate to choose the interbank rate and thus implement the optimal allocation.

To respond to aggregate liquidity shocks, it is necessary for central banks to inject

liquid assets into banks when such a shock occurs. This can be done, for example, if some goods are taxed by a public authority and stored. If the shock occurs, the public authority injects the stored goods into the banking system. If the shock does not occur, the goods are kept and redistributed to patient agents at a later date. A central bank, or some other public institution, can implement this policy.

9 Appendix A: Generalization to N states

Consider a generalization of the baseline model (without runs or liquidation of assets) with N idiosyncratic states $i_1, ..., i_N \geq 0$. We assume $i_1 = 0$, $\lambda^{i_n H} = \overline{\lambda} + i_n \varepsilon$, and $\lambda^{i_n L} = \overline{\lambda} - i_n \varepsilon$, where $i_n \in \{i_1, ..., i_N\}$. The probability of i_n is ρ_n, $\sum_{n=1}^{N} \rho_n = 1$.

A bank's problem is thus

$$\begin{aligned} \max_{\alpha \in [0,1]; c_1, \{\beta^{ij}\}_{i \in \mathcal{I}, j \in \mathcal{J}} \geq 0} \quad & \overline{\lambda} u(c_1) + \sum_{n=1}^{N} \rho_n [\tfrac{1}{2}(1 - \lambda^{i_n H}) u(c_2^{i_n H}) + \tfrac{1}{2}(1 - \lambda^{i_n L}) u(c_2^{i_n L})] \\ \text{s.t.} \quad & \lambda^{i_n j} c_1 \leq 1 - \alpha + \beta^{i_n j} + f^{i_n j} \\ & (1 - \lambda^{i_n j}) c_2^{i_n j} \leq \alpha r - \beta^{i_n j} - f^{i_n j} \iota^{i_n} \\ & \text{for } i_n \in \{i_1, ..., i_N\},\ j \in \mathcal{J}. \end{aligned}$$

The first-order conditions with respect to α and c_1 are, respectively,

$$\sum_{n=1}^{N} \rho_n [\tfrac{1}{2} u'(c_2^{i_n H}) + \tfrac{1}{2} u'(c_2^{i_n L})] \iota^{i_n} = \sum_{n=1}^{N} \rho_n [\tfrac{1}{2} u'(c_2^{i_n H}) + \tfrac{1}{2} u'(c_2^{i_n L})] r \tag{25}$$

$$u'(c_1) = \sum_{n=1}^{N} \rho_n [\tfrac{\lambda^{i_n H}}{2\overline{\lambda}} u'(c_2^{i_n H}) + \tfrac{\lambda^{i_n L}}{2\overline{\lambda}} u'(c_2^{i_n L})] \iota^{i_n}. \tag{26}$$

By the same logic as in the case with two states, the interest rate in the interbank market should be equal to $\frac{c_2^*}{c_1^*}$ whenever $i_n > 0$ in order to facilitate risk-sharing between banks. Without loss of generality, assume that $i_n > 0$ for all $n \geq 2$. Then we have $\iota^{i_n} = \frac{c_2^*}{c_1^*}$ and $c_2^{i_n H} = c_2^{i_n L} = \frac{\alpha r}{1 - \overline{\lambda}}$ for all $n \geq 2$. Let $\rho = \sum_{n=2}^{N} i_n$, and then we can write interest rate ι^{i_1} as

$$\iota^{i_1} = r + \frac{\rho(r - \frac{c_2^*}{c_1^*})}{1 - \rho}, \tag{27}$$

which is equal to $\iota^0 = \iota^{0^*}$ in the two-state baseline model.[5]

[5] We can show that if there is no state with a zero-size shock, then a first best equilibrium does not exist because an equilibrium requires an interest rate of $l^i > \frac{c_2^*}{c_1^*}$ for at least one idiosyncratic state i, which is then always distortionary. If the baseline model is modified such that with two idiosyncratic states $0 < i_0 < i_1$, we can show that there is a constrained-efficient equilbrium with $l^{1^*} < l^{i_1} < r < l^{i_0} < l^{0^*}$, which is chosen by the central bank.

10 Appendix B: Monetary policy with nominal rates

We expand the real model to allow for nominal interbank lending rates. With nominal fiat interest rates, the central bank can explicitly enforce its target for the interbank rate, in order to actively select the rational expectations equilibrium. The central bank offers to borrow and lend to banks any amount of nominal, fiat money at the central bank's policy rate at date 1, which ensures that the interbank market rate equals the central bank's policy rate. The equilibrium and allocation of the nominal rate model is equivalent to the real rate model.

10.1 Nominal rate model extension

The extension of the model to include nominal rates is based on Skeie (2008). A nominal unit of account, inside money and a goods market with firms are added to the model of banks with real deposits. To establish a fiat nominal unit of account, the central bank offers at date 0 to buy or sell goods to the extent feasible for fiat currency (equivalent to central bank reserves) at a fixed nominal price $P_0 = 1$. After date 0, the central bank does not set the price of goods and does not offer to buy or sell goods. At date 0, each bank makes a loan to a firm. The firm buys the good from the bank's unit continuum of consumers, and consumers deposit in the bank. All the transactions at date 0 are paid for in the amount of one nominal unit of account. These nominal payments can be called "inside money," and are payable simultaneously. Inside money payments are "settled" in currency, which is defined as inside money payments being netted and any outstanding inside money amount due being paid in currency. Since zero currency is held by each agent, the individual budget constraint for each of the banks, firms and consumers requires that the net inside money payments of each party must net to zero at date 0.

Again, to simplify the exposition, we assume that the fraction of impatient depositors is always $\overline{\lambda}$. Each bank lends to its firm for loan repayments of nominal amounts $(1-\alpha) K_1$ and αK_2^i payable in inside money at dates 1 and 2, respectively. Uppercase variables denote nominal values and lowercase variables denote real values. The firm buys the good from consumers for price $P_0 = 1$. The firm invests α and stores $1-\alpha$ of the good, where α is chosen and can be enforced by the bank.[6] Consumers deposit in their bank for a

[6] If the bank could not enforce the firm's storage, the bank could alternatively buy and store $1-\alpha$

demand deposit contract that pays in inside money a nominal consumption amount of either $C_1 \geq 1$ if withdrawn at date 1 or $C_2^{ij} \geq 1$ if withdrawn at date 2. Although no currency circulates, the central bank's offer to trade with currency establishes the nominal unit of account for transactions with bank inside money. This is equivalent to Skeie (2008), where currency rather than inside money transactions occur at date 0.

In each period $t = 1, 2$, payments are made simultaneously among banks with either currency or inside money that is settled with currency. At date 1, λ^{ij} early consumers of bank j withdraw to buy goods from a firm in the goods market. At date 2, $1 - \lambda^{ij}$ late consumers withdraw from banks to buy goods. The representative firm repays loans and banks borrow or lend inside money if needed on the interbank market or currency from the central bank.

The bank's budget constraints from the real model (1) and (2) are replaced by budget constraints for nominal payments:

$$\text{s.t.} \qquad \lambda^{ij} C_1 = (1-\alpha)K_1 + M_f^{ijD} + M_o^{ijD}, \qquad \forall\ i \in \mathcal{I},\ j \in \mathcal{J}, \tag{28}$$

$$(1-\lambda^{ij})C_2^{ij} = \alpha K_2^i - M_f^{ijD} R_f^i - M_o^{ijD} R_o^i, \qquad \forall\ i \in \mathcal{I},\ j \in \mathcal{J}, \tag{29}$$

respectively, where bank j's demand to borrow from other banks is M_f^{ijD} and from the central bank (in currency) is M_o^{ijD}, and where R_f^i and R_o^i are the returns on interbank loans and central bank loans, respectively. The notation 'M' represents money (inside money or currency), subscript 'f' represents the fed funds interbank market, and subscript 'O' represents open market operations. R_f^i is the interbank market rate, which is determined in equilibrium. At date 1, the central bank targets R_f^i by choosing its policy rate R_o^i at which it offers to borrow and lend to banks an unlimited amount. Specifically, the central offers to supply a loan of $M_o^{ijS}(R_o^i) \in (-\infty, \infty)$ to bank j at rate R_o^i, where $M_o^{ijS}(R_o^i)$ is a correspondence. The central does not have a budget constraint to equate its borrowing and lending of central bank currency, since it can create and destroy currency as needed. The central bank's lending supply is perfectly elastic at its chosen rate R_o^i. The way in which we model the central bank offering to borrow and lend at a single policy rate is similar to open market operations in practice. Many central banks in essence offer to borrow and lend a perfectly elastic amount of funds at a chosen rate to target the interbank rate at

goods, sell them at date, and lend α to the firm without any storage requirements. Results of the model would be unchanged.

which banks lend uncollateralized to each other. Open market operations lending is often collateralized in practice, as in the form of repos against government securities in the case of the Federal Reserve. We abstract from collateralization since there is no risk of loss or default.

Consumers buy goods from firms at date $t = 1, 2$ in a Walrasian market using inside money as numeraire. Consumption for early and late consumers is

$$c_1(P) = \frac{C_1}{P_1} \tag{30}$$

$$c_2^{ij}(P) = \frac{C_2^{ij}}{P_2^i}, \tag{31}$$

where P_t^i is the nominal price of goods at date $t = 1, 2$ and $P \equiv (P_1, P_2^i)$ is a vector. We consider only $P_t^i \in (0, \infty)$, which is for simplicity and does not effect the results. Consumers' aggregate demand is given by

$$q_1^D(P) = \frac{\frac{1}{2}(\lambda^{ih} + \lambda^{il})C_1}{P_1} \tag{32}$$

$$q_2^D(P) = \frac{\frac{1}{2}[(1 - \lambda^{ih}) + (1 - \lambda^{il})]C_2^{ij}}{P_2^i}. \tag{33}$$

The representative firm submits a supply schedule $q_t^{iS}(P_t^i)$ for the goods market. The firm's optimization is to maximize profits:

$$\max_{q_1^{iS}, q_2^{iS} \geq 0} \quad 1 - \alpha + \alpha r - q_1^{iS} - q_2^{iS} \tag{34a}$$

$$\text{s.t.} \quad q_1^{iS} \leq 1 - \alpha \tag{34b}$$

$$q_2^{iS} \leq 1 - \alpha + \alpha r - q_1^{iS} \tag{34c}$$

$$q_1^{iS} \geq \frac{(1 - \alpha) K_1}{P_1} \tag{34d}$$

$$q_2^{iS} \geq \frac{\alpha K_2^i}{P_2^i}. \tag{34e}$$

The objective function (34a) is the profit in goods that the firm consumes at date 2. Constraints (34b) and (34c) are the maximum amounts of goods that can be sold at dates 1 and 2, respectively. Constraints (34d) and (34e) are the firm's budget constraints to repay its loan at date 1 and date 2, respectively.

The bank's demand for borrowing on the interbank market can be solved for from equation (28) as

$$M_f^{ijD} = \lambda^{ij} C_1 - (1 - \alpha) K_1 - M_o^{ijD}. \tag{35}$$

Substituting for M_f^{ijD} from equation (35) into equation (29) and rearranging, we find that bank j pays withdrawals to late consumers the amount

$$C_2^{ij} = \frac{\alpha K_2^i - [\lambda^{ij} C_1 - (1-\alpha) K_1] R_f^i + (R_f^i - R_o^i) M_o^{ijD}}{1 - \lambda^{ij}}. \tag{36}$$

The bank's optimization problem (4) is replaced by

$$\begin{aligned} \max_{\alpha \in [0,1]; C_1 \geq 0; \{M_o^{ijD}\}_{i,j} \in \mathbb{R}} \quad & E[U], \\ \text{s.t.} \quad & (36) \end{aligned} \tag{37}$$

where $c_1(P)$ and $c_2^{ij}(P)$ are given by (30) and (31), respectively.

An *equilibrium* is defined as goods market prices and quantities (P, q_1, q_2), deposit and loan returns and quantities $\{C_1, R_f^i, M_o^{ij}\}_{i,j}$, and investment (α) that solve goods market clearing conditions

$$q_t^D(P) = \tfrac{1}{2}[q_t^{hS}(P) + q_t^{lS}(P)] \text{ for } t = 1, 2,$$

and interbank market clearing condition

$$M_f^{hD}(M_o^{hD}) + M_f^{lD}(M_o^{lD}) = 0, \tag{39}$$

where $\{\alpha, C_1, M_o^{ij}\}_{i,j}$ is a solution to bank j's optimization (37); $\{q_t^D(P)\}_{t=1,2}$ is given by the consumers' aggregate demand (32) and (33), and and $(q_1^{iS}(P), q_2^{iS}(P))$ is a solution to the firm's optimization (34).

10.2 Nominal rate results

The results of the nominal model are equivalent to those of the real model, with the addition that the central bank can choose its policy rate to target the interbank rate. The first order conditions for bank j's optimization (37) with respect to α, c_1 and M_o^{ijD} are

$$E[\frac{K_2^i}{P_2^i} u'(c_2^{ij})] = E[\frac{K_1}{P_2^i} R_f^i u'(c_2^{ij})], \qquad \forall\ i \in \mathcal{I} \tag{40}$$

$$E[\frac{1}{P_1} u'(c_1)] = E[\frac{\lambda^{ij} R_f^i}{\overline{\lambda} P_2^i} u'(c_2^{ij})], \qquad \forall\ i \in \mathcal{I} \tag{41}$$

$$R_f^i = R_o^i, \qquad \forall\ i \in \mathcal{I}, \tag{42}$$

respectively. Loan returns are set according to a competitive loan market as

$$K_1 = P_1 \tag{43}$$

$$K_2^i = r P_2^i, \tag{44}$$

such that the real returns $\frac{K_1}{P_1} = 1$ and $\frac{K_2^i}{P_2^i} = r$ equal the marginal product of capital for their respective terms and firms make zero profits in equilibrium. Substituting for K_t^i from equations (43) and (44), conditions (40) and (41) can be written as

$$E[u'(c_2^{ij})] = E[\frac{R_f^i}{P_2^i/P_1}u'(c_2^{ij})] \tag{45}$$

$$E[u'(c_1)] = E[\frac{\lambda^{ij}}{\overline{\lambda}}\frac{R_f^i}{P_2^i/P_1}u'(c_2^{ij})]. \tag{46}$$

Condition (42) states that because of arbitrage, the interbank rate R_f^i equals the central bank's policy rate R_o^i. The real interbank rate equals the nominal rate divided by nominal goods price inflation between dates 1 and 2:

$$\iota^i = \frac{R_f^i}{P_2^i/P_1}, \tag{47}$$

which implies that the first order conditions for the nominal model, equations (45) and (46), and for the real model, equations (13) and (14), are equivalent. The central bank can target any real interbank lending rate ι^i at date 1 by choosing

$$R_o^i = \frac{P_2^i}{P_1}\iota^i,$$

subject to satisfying the date 0 first order conditions for ι^0 and ι^1. In particular, the central bank can implement the first best allocation by choosing

$$R_o^i = \overline{R}_o^i \equiv \frac{P_2^i}{P_1}\bar{\iota}^i. \tag{48}$$

Proposition 5. *The central bank can choose $R_o^i = \overline{R}_o^i$, and there exists a unique equilibrium with first best allocation $\alpha = \alpha^*$, $c_1 = c_1^*$ and $c_2^{ij} = c_2^*$.*

Proof. Equilibrium prices and quantities satisfy

$$P_1 = \frac{\overline{\lambda}C_1}{q_1} \tag{49}$$

$$P_2^i = \frac{(1-\overline{\lambda})C_2^{ij}}{q_2}. \tag{50}$$

The constraints in the firm's optimization (34) bind, which gives

$$q_1 = 1 - \alpha \tag{51}$$

$$q_2 = \alpha r. \tag{52}$$

Substitution for quantities and prices from (49) - (52) into (30) and (31),

$$c_1 = \frac{1-\alpha}{\overline{\lambda}} \tag{53}$$

$$c_2^{0j} = \frac{\alpha r}{1-\overline{\lambda}}. \tag{54}$$

To find C_1, substituting for M_f^{ijD} from (35) into the market clearing condition (39) and simplifying gives

$$C_1 = \frac{(1-\alpha)\,K_1}{\overline{\lambda}} + \frac{M_o^{ihD} + M_o^{ilD}}{2\overline{\lambda}}. \tag{55}$$

Substituting from (55) for C_1 into (35) and simplifying gives the demand for interbank borrowing by bank j as

$$M_f^{ijD} = (\frac{\lambda^{ij}}{\overline{\lambda}} - 1)\,(1-\alpha)\,K_1 + \frac{\lambda^{ij}}{\overline{\lambda}}(M_o^{ihD} + M_o^{ilD}) - M_o^{ijD}.$$

Rearranging, aggregate bank borrowing is

$$M_f^{ijD} + M_o^{ijD} = (1-\alpha)\,K_1(\frac{\lambda^{ij}}{\overline{\lambda}} - 1) + \frac{\lambda^{ij}}{\overline{\lambda}}(M_o^{ihD} + M_o^{ilD}), \tag{56}$$

Using (56), we can show that

$$(M_f^{ihD} + M_f^{ilD}) + (M_o^{ihD} + M_o^{ilD}) = 2(M_o^{ihD} + M_o^{ilD}). \tag{57}$$

By market clearing equation (39), aggregate net interbank borrowing is zero, $M_f^{ihD} + M_f^{ilD} = 0$, which by equation (56) implies $(M_o^{ihD} + M_o^{ilD}) = 2(M_o^{ihD} + M_o^{ilD})$. Hence, $(M_o^{ihD} + M_o^{ilD}) = 0$. Aggregate net borrowing from the central bank is zero in equilibrium. The central bank lends zero net supply of currency to the market. While bank j aggregate net borrowing from the interbank market and the central bank is determined by equation (56) as $M_f^{ijD} + M_o^{ijD} = (1-\alpha)\,K_1(\frac{\lambda^{ij}}{\overline{\lambda}} - 1)$, the individual components M_f^{ijD} and M_o^{ijD} are not determined. The central bank does not need to lend to any banks in equilibrium. Lending by the central bank is equivalent and a substitute for interbank lending.

Substition into (36) for K_t from (43) and (44), for R_o^i from (42), for R_o^i from (48), for $1-\alpha = \overline{\lambda}c_1$ from (53), and for $C_1 = \frac{P_1 q_1}{\overline{\lambda}} = \frac{P_1(1-\alpha)}{\overline{\lambda}}$ from (49) and (51), and rearranging gives

$$c_2^{ij} = \frac{C_2^{ij}}{P_2^i} = \frac{\alpha r - (\lambda^{ij} - \overline{\lambda})c_1\overline{\iota}^i}{1-\lambda^{ij}},$$

which is identical to c_2^{ij} in the real model given by equation (16). The bank has an optimization identical to that in the real model and chooses $\alpha = \alpha^*$. Hence, the equilibrium is identical to that of the real model and the allocation is $c_1 = c_1^*$ and $c_2^{ij} = c_2^*$. ■

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